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RESEARCH ARTICLE

PRIME LABELING OF SPECIAL GRAPHS

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ABSTRACT

Prime labeling is the most interesting category of graph labeling with various applications. A graph $G = (V(G), E(G))$ with $|V(G)|$ vertices are said to have prime labeling if its vertices are labeled with distinct positive integers $1,2,3, \dots, |v|$ such that for each edge $uv \epsilon E(G)$ the labels assigned to u and v are relatively prime, where $V(G)$ and $E(G)$ are vertex set and edge set of G, respectively. Therefore, the graph G has a prime labeling whenever any of two adjacent vertices can be labeled as two relative prime numbers and is called a prime graph. In our work, we focus on the prime labeling method for newly constructed graphs obtained by replacing each edge of a star graph $K_{1,n}$ by a complete tripartite graph $K_{1,m,1}$ for $m = 2, 3, 4,$ and 5, which are prime graphs. In addition to that, investigate another type of simple undirected finite graphs generalized by using circular ladder graphs. These new graphs obtained by attaching $K_{1,2}$ at each external vertex of the circular ladder graph CL_n and proved that the constructed graphs are prime graphs when $n \geq 3$ and $n \not\equiv 1 (mod\ 3)$. Finally, focus on another particular type of simple undirected finite graph called a scorpion graph, denoted by $S_{(2p,2q,r)}$. The Scorpion graph gets its name from shape, which resembles a scorpion, having $2p + 2q + r$ vertices $(p \geq 1,\ q \geq 2, r \geq 2)$ are placed in the head, body, and tail respectively. To prove that the scorpion graph has prime labeling, we used two results that have already been proved for ladder graphs.

Keywords: Cyclic Ladder graph, Greatest Common Divisor, Star Graph, Tripartite Graph, Prime Labeling

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1. INTRODUCTION

Graph labeling is a prominent research area in Graph theory, and there are considerable number of open problems and literature are available for various types of graphs. Rosa introduced the theory behind graph labeling in the 1960s. Roger Entringer introduced the concept of prime labeling, and around 1980 he conjectured that all trees have prime labeling, which is not settled until today. Tout. et al. introduced the method of prime graph labeling in 1982. In prime graph labeling, distinct positive integers are assigned to the vertices which are less than or equal to the number of vertices in the graph such that labels of adjacent vertices are relatively prime. Edge labeling is another particular area in graph labeling and was introduced by Deretsky, Lee, and Mitchem in 1991. Throughout this paper, we considered the vertex prime labeling for a new type of graphs. Many researchers had proved that various types of graphs are prime graphs. The graph G with vertex set $V(G)$ and edge set $E(G)$ with $|V(G)|$ vertices is said to have prime labeling if there exists a bijective mapping $f : V(G) \rightarrow \{1,2,3, \dots, |V(G)|\}$ such that for each edge $e = uv$ in $E(G)$, $f(u)$ and $f(v)$ are relatively prime and such a graph is called a Prime graph. In our previous work, prime labeling of Crab graph, Roach graph, Centerless double wheel graph, and a Complete tripartite graph $K_{1,m,1}$ for $m = 2,3$ have been discussed. In this paper, we consider prime labeling of newly constructed graphs obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,m,1}$ for $m = 2,3,4$ and 5, Stripe Blade Fan Graph $(F_{1,3,m})$, simple undirected finite graph obtained by taking the union of the star graph $K_{1,2}$ and the circular ladder graph CL_n and the Scorpion graph. We provide a brief summary of definitions and theorems which are necessary for the present investigations.

Definition 1 (Complete Tripartite Graph): A *complete tripartite graph* is a graph of the form $K_{p,q,r}$ which has p vertices in one partite set, q vertices in another partite set, and r vertices in the remaining partite set where $p, q, r \geq 1$ in which each vertex in one partite set is adjacent to all the vertices in the other two partite sets.

Definition 2 (Stripe Blade Fan Graph -$F_{1,3,n}$): A *Stripe Blade Fan graph* is a graph obtained by star graph $K_{1,3}$ by replacing edge by a complete bipartite graph $K_{1,n,1}$ which has $3n + 4$ vertices and $6(n - 1)$ edges and is denoted by $F_{1,3,m}$.

Definition 3 (Circular Ladder Graph -CL_n): Let P_n denotes the path on n vertices, then the Cartesian product $P_n \times P_m$, where $n \geq m$, is called a grid graph. If $m = 2$, then the graph is called a ladder graph. A circular ladder graph (Prism graph) denoted by CL_n. n −Prism graph has $2n$ vertices and $3n$ edges.

Theorem 1 [1]: If $n + 1$ is prime, then $P_n \times P_2$ has a prime labeling. Moreover, this prime labeling can be realized with top row labels from left to right, $1, 2, \dots, n$, and bottom row labels from left to right, $n + 2,\ n + 3,\ \dots,\ 2n,\ n + 1$.

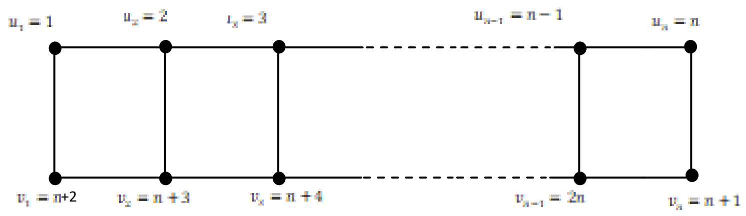

Figure 1: The vertex labeling of the ladder graph

Theorem 2. [1]: $P_n \times P_2$ has a consecutive cyclic prime labeling with the value 1 assigned to the vertex u_1 if and only if $2n + 1$ is prime.

2. MATERIAL AND METHODS

In our research, prime labeling of some special types of graphs were given by the following theorems.

Theorem 1: The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,2,1}$ is a prime graph, where $n \geq 1$.

Proof: Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,2,1}$, where $n \geq 1$. Labeling the vertices of $K_{1,n}$ as $u_0, u_1, u_2, \dots, u_n$ with u_0 be the center vertex and every edge $u_0 u_i$ of $K_{1,n}$ is replaced by $K_{1,2,1}$ for $1 \leq i \leq n$, where $n \geq 1$.

Then, the new vertex set is $V(H) = \{u_0, u_i, u_{i1}, u_{i2}\}$, for $1 \leq i \leq n$.

Also, the new edge set is $E(H) = \{u_0 u_i, u_0 u_{i1}, u_{i1} u_i, u_0 u_{i2}, u_{i2} u_i\}$, for $1 \leq i \leq n$.

So $|V(H)| = 3n + 1$.

Define a function $f: V(H) \to \{1,2,3, \dots, 3n + 1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = 3i \text{ for } 1 \leq i \leq n.$$

$$f(u_{i1}) = 3i - 1 \text{ for } 1 \leq i \leq n.$$

$$f(u_{i2}) = 3i + 1 \text{ for } 1 \leq i \leq n.$$

Note that,

$$\gcd\big(f(u_0), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1 \text{ for} 1 \leq i \leq n.$$

$$\gcd\big(f(u_0), f(u_{i1})\big) = \gcd\big(1, f(u_{i1})\big) = 1 \text{ for} 1 \leq i \leq n.$$

$$\gcd\big(f(u_0), f(u_{i2})\big) = \gcd\big(1, f(u_{i2})\big) = 1 \text{ for} 1 \leq i \leq n.$$

Consider, for $1 \leq i \leq n$.,

$$\gcd\big(f(u_{i1}),f(u_i)\big) = \gcd(3i-1,3i\ ) = 1 \text{ (consecutive positive integers)}$$

$$\gcd\big(f(u_{i2}),f(u_i)\big) = \gcd(3i+1,3i\ ) = 1 \text{(consecutive positive integers)}$$

Therefore, vertex labels are distinct. Thus, labeling defined above gives a prime labeling for G. Therefore, G is a prime graph.

Example 1

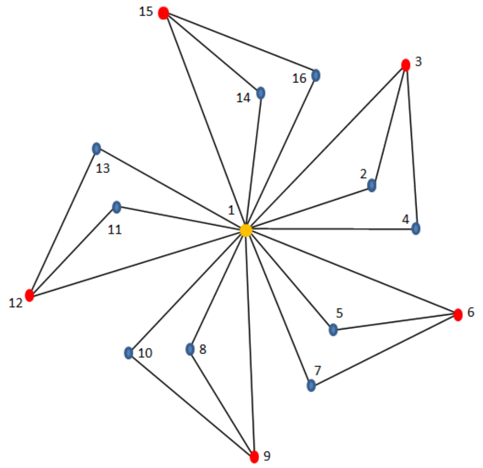

Figure 2: Prime labeling of the graph obtained by replacing every edge of a star graph $K_{1,5}$ by $K_{1,2,1}$

Theorem 2: The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,3,1}$ is a prime graph, where $n \geq 1$.

Proof: This proof is similar to the proof of theorem 1. However, the function is defined as follows for these types of graphs. Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,3,1}$, where $n \geq 1$. Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \dots, u_n$ with u_0 as the vertex at the center and every edge u_0u_i of $K_{1,n}$ is replaced by $K_{1,3,1}$ for $1 \leq i \leq n$, where $n \geq 1$.

Then, the new vertex set is

$$V(H) = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}\} \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

Also, the new edge set is

$$E(H) = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i\}, \text{ for } 1 \leq i \leq n, n \geq 1.$$

So, $|V(H)| = 4n+1$, where $n \geq 1$.

Define a function $f: V(H) \to \{1,2,3,\dots,4n+1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = 4i-1 \text{ for} 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i1}) = 4i-2 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$f(u_{i2}) = 4i$ for $1 \leq i \leq n$, where $n \geq 1$.

$f(u_{i3}) = 4i + 1$ for$1 \leq i \leq n$, where $n \geq 1$.

Example 2 The prime labeling of the graph obtained by replacing every edge of a star graph $K_{1,4}$ by $K_{1,3,1}$ using labeling appears in the following figure.

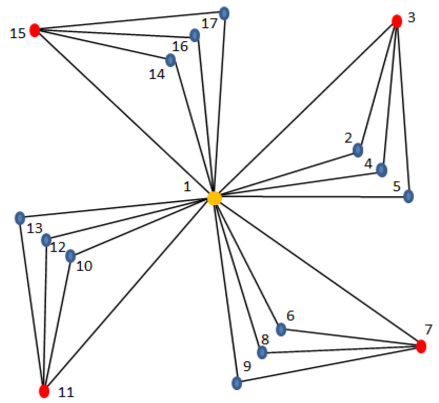

Figure 3: Prime labeling of the graph obtained by replacing every edge of a star graph $K_{1,4}$ by $K_{1,3,1}$

Theorem 3: The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,4,1}$ is a prime graph, where $n \geq 1$.

Proof: let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,4,1}$, where $n \geq 1$. Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \dots, u_n$ with u_0 as the vertex at the center and every edge u_0u_i of $K_{1,n}$ replaced by $K_{1,4,1}$ for $1 \leq i \leq n$, where $n \geq 1$. Then, the new vertex set is

$$V(H) = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}, u_{i4}\} \text{ for } 1 \leq i \leq n, n \geq 1.$$

Also, the new edge set is,

$$E(H) = \{u_0u_i,\ u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i, u_0u_{i4}, u_{i4}u_i\}, \text{ for} 1 \leq i \leq n,$$

where $n \geq 1$.

So, $|V(H)| = 5n + 1$, where $n \geq 1$.

Define a labeling function $f: V(H) \to \{1,2,3, \dots, 5n + 1\}$ as follows:

$f(u_0) = 1$ for all i

$$f(u_i) = \begin{cases} 5i, & i \equiv 1 \ mod(6) \\ 5i - 1, & i \equiv 0,2 \ mod(6) \\ 5i - 2, & i \equiv 3,5 \ mod(6) \\ 5i - 3, & i \equiv 4 \ mod(6) \end{cases}$$

$$f(u_{i2}) = \begin{cases} 5i-2, & i \not\equiv 3,4,5 \ mod(6) \\ 5i-1, & i \equiv 3,4,5 \ mod(6) \end{cases}$$

$$f(u_{i3}) = \begin{cases} 5i-1, & i \equiv 1 \ mod(6) \\ 5i, & i \not\equiv 1 \ mod(6) \end{cases}$$

$$f(u_{i4}) = 5i+1, \quad 1 \le i \le n$$

$$f(u_{i1}) = \begin{cases} 5i-3, & i \not\equiv 4 \ mod(6) \\ 5i-2, & i \equiv 4 \ mod(6) \end{cases}$$

Note that,

$\gcd(f(u_0), f(u_i)) = \gcd(1, f(u_i)) = 1$, for $1 \le i \le n$, where $n \ge 1$.

$\gcd(f(u_0), f(u_{i1})) = \gcd(1, f(u_{i1})) = 1$, for $1 \le i \le n$, where $n \ge 1$.

$\gcd(f(u_0), f(u_{i2})) = \gcd(1, f(u_{i2})) = 1$, for $1 \le i \le n$, where $n \ge 1$.

$\gcd(f(u_0), f(u_{i3})) = \gcd(1, f(u_{i3})) = 1$ for $1 \le i \le n$, where $n \ge 1$.

$\gcd(f(u_0), f(u_{i4})) = \gcd(1, f(u_{i4})) = 1$ for $1 \le i \le n$, where $n \ge 1$.

Case 1: Assume $i \equiv 1 \ mod(6)$,then,$f(u_i) = 5i$, note that,

$\gcd(f(u_i), f(u_{i1})) = \gcd(5i, 5i-3) = 1$ (not multiple of 3 and differ by 3)

$\gcd(f(u_i), f(u_{i2})) = \gcd(5i, 5i-2) = 1$ (consecutive odd numbers)

$\gcd(f(u_i), f(u_{i3})) = \gcd(5i, 5i-1) = 1$(consecutive positive numbers)

$\gcd(f(u_i), f(u_{i4})) = \gcd(5i, 5i+1) = 1$ (consecutive positive numbers)

Case 2: Assume $i \equiv 0,2 \ mod(6)$, so that $f(u_i) = 5i-1$, note that,

$\gcd(f(u_i), f(u_{i1})) = \gcd(5i-1, 5i-3) = 1$ (consecutive odd numbers)

$\gcd(f(u_i), f(u_{i2})) = \gcd(5i-1, 5i-2) = 1$ (consecutive positive numbers)

$\gcd(f(u_i), f(u_{i3})) = \gcd(5i-1, 5i) = 1$ (consecutive positive numbers)

$\gcd(f(u_i), f(u_{i4})) = \gcd(5i-1, 5i+1) = 1$ (consecutive odd numbers)

Case 3: Assume $i \equiv 3,5 \ mod(6)$, so that $f(u_i) = 5i-2$, note that,

$\gcd(f(u_i), f(u_{i1})) = \gcd(5i-2, 5i-3) = 1$ (consecutive positive numbers)

$\gcd(f(u_i), f(u_{i2})) = \gcd(5i-2, 5i-1) = 1$ (consecutive positive numbers)

$\gcd(f(u_i), f(u_{i3})) = \gcd(5i-2, 5i) = 1$ (consecutive odd numbers)

$\gcd(f(u_i), f(u_{i4})) = \gcd(5i-2, 5i+1) = 1$ (are not multiple of 3 and differ by 3)

Case 4: Assume $i \equiv 4 \ mod(6)$, so that $f(u_i) = 5i - 3$, note that,

$$\gcd\big(f(u_i), f(u_{i1})\big) = \gcd(5i - 3, 5i - 2) = 1 \text{ (consecutive positive numbers)}$$

$$\gcd\big(f(u_i), f(u_{i2})\big) = \gcd(5i - 3, 5i - 1) = 1 \text{ (consecutive odd numbers)}$$

$$\gcd\big(f(u_i), f(u_{i3})\big) = \gcd(5i - 3, 5i) = 1 \text{ (not multiple of 3 and differ by 3)}$$

$$\gcd\big(f(u_i), f(u_{i4})\big) = \gcd(5i - 3, 5i + 1) = 1 \text{ (odd integers that differ by 4)}$$

Clearly, vertex labels are distinct. Thus, the labeling method defined above gives a prime labeling for G. Therefore, G is a prime graph.

Theorem 4: The graph G obtained by replacing every edge of a star graph $K_{1,n}$ by the tripartite graph $K_{1,5,1}$ is a prime graph, where $n \geq 1$.

Proof: Let G be a graph obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,5,1}$, where $n \geq 1$. Let the vertices of $K_{1,n}$ be $u_0, u_1, u_2, \dots, u_n$ with u_0 as the center vertex and every edge u_0u_i of $K_{1,n}$ replaced by $K_{1,5,1}$ for $1 \leq i \leq n$ where $n \geq 1$. Then, the new vertex set is

$$V(H) = \{u_0, u_i, u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5}\} \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

Also, the new edge set is,

$$E(H) = \{u_0u_i, u_0u_{i1}, u_{i1}u_i, u_0u_{i2}, u_{i2}u_i, u_0u_{i3}, u_{i3}u_i, u_0u_{i4}, u_{i4}u_i, u_0u_{i5}, u_{i5}u_i\},$$

for $1 \leq i \leq n$,where $n \geq 1$.

So,$|V(H)| = 6n + 1$,where $n \geq 1$.

Define a function $f: V(H) \to \{1,2,3, \dots, 6n + 1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = 6i - 1 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i1}) = 6i - 4 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i2}) = 6i - 3 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i3}) = 6i - 2 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i4}) = 6i \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

$$f(u_{i5}) = 6i + 1 \text{ for } 1 \leq i \leq n, \text{ where } n \geq 1.$$

This proof is similar to the proof of Theorem 1.

Example 3 When $n = 3$; the resultant graph is called a Stripe Blade Fan Graph $(F_{1,3,m})$. The following figure shows stripe blade fan graphs for $m = 4$ and $m = 5$.

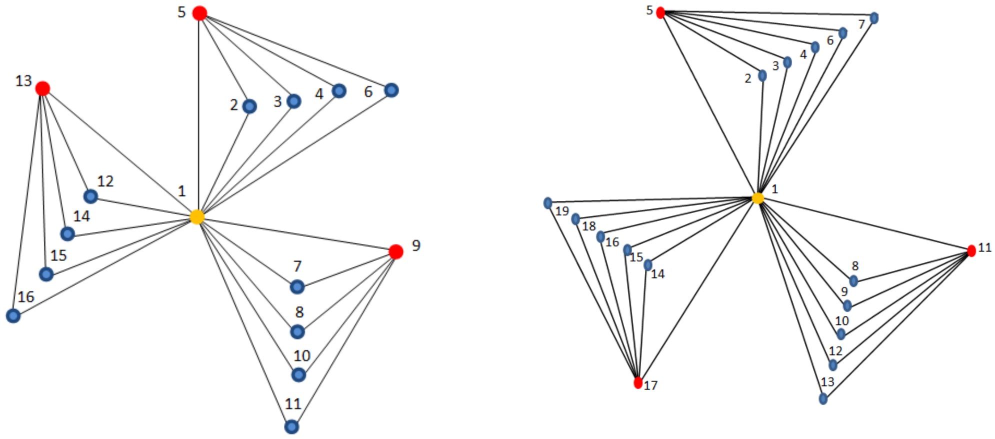

Figure 4: Prime labeling of graphs obtained by replacing every edge of a star graph $K_{1,3}$ by $K_{1,4,1}$ and $K_{1,5,1}$.

Theorem 5: The graph G obtained by attaching $K_{1,2}$ at each external vertex of CL_n is a prime graph, when $n \geq 3$ and $n \not\equiv 1(mod\ 3)$.

Proof: Let $u_1, u_2, u_3, \dots, u_n$ be the internal vertices in the circular ladder graph and $v_1, v_2, v_3, \dots, v_n$ be the external vertices in the circular ladder graph. Let v_i, v_{i1} and v_{i2} be the vertices of i^{th} copy of $K_{1,2}$ in which v_i is the central vertex, where $1 \leq i \leq n, n \geq 3$ and $n \not\equiv 1(mod\ 3)$. Now, the vertex set of G is

$$V(G) = \{u_1, u_2, u_3, \dots, u_n, v_1, v_2, v_3, \dots, v_n, v_{i1}, v_{i2}\}.$$

The edge set of G is

$$E(G) = \{ u_i u_{i+1}, u_1 u_n, v_i v_{i+1}, v_1 v_n, v_i v_j; 1 \leq i \leq n, n \geq 3, n \not\equiv 1(mod\ 3), j = 1,2 \}.$$

Then, $|V(G)| = 4n$ and $|E(G)| = 5n$.

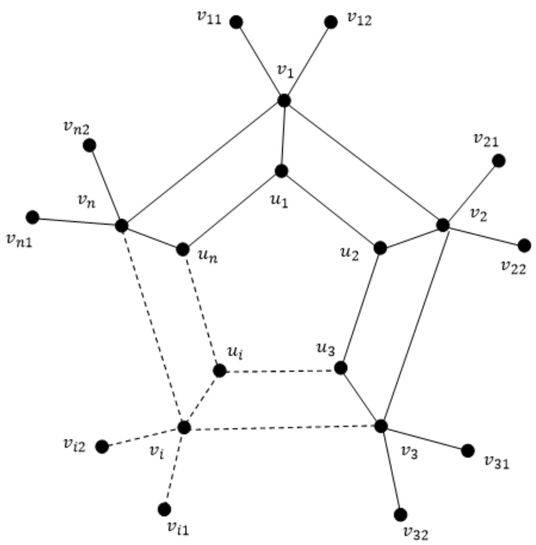

Figure 5: The graph obtained by attaching $K_{1,2}$ at each external vertex of the circular ladder graph

Define a function $f: V(G) \rightarrow \{1,2,3, \dots, 4n\}$ by

$f(u_i) = 4i - 3$ for $1 \leq i \leq n$,

$f(v_i) = 4i - 1$ for $1 \leq i \leq n$,

$f(v_{i1}) = 4i - 2$ for $1 \leq i \leq n$,

$f(v_{i2}) = 4i$ for $1 \leq i \leq n$.

Consider,

$\gcd(u_i, u_{i+1}) = \gcd(4i - 3, 4i + 1) = 1$, where $1 \leq i \leq n - 1$ (are not multiple of 2 and differ by 4);

$\gcd(u_i, v_i) = \gcd(4i - 3, 4i - 1) = 1$, $1 \leq i \leq n$ (consecutive odd numbers);

$\gcd(v_i, v_{i+1}) = \gcd(4i - 1, 4i + 3) = 1$, where $1 \leq i \leq n$ (are not multiple of 2 and differ by 4);

$\gcd(v_i, v_{i1}) = \gcd(4i - 1, 4i - 2) = 1$, where $1 \leq i \leq n$ (consecutive positive integers);

$\gcd(v_i, v_{i2}) = \gcd(4i - 1, 4i) = 1$, where $1 \leq i \leq n$ (consecutive positive integers);

$\gcd(u_1, u_n) = \gcd(1, u_n) = 1$, where $n \geq 3$ and $n \not\equiv 1 (mod\ 3)$ (clearly);

$\gcd(v_1, v_n) = \gcd(3, 4n - 1) = \gcd(3, 4n + 2) = 1$, where $n \geq 3$ and $n \not\equiv 1 (mod\ 3)$ (3 is odd and $4n + 2$ is even).

Clearly, vertex labels are distinct. Thus, the above labeling method gives a prime labeling for G. Hence, G is a prime graph.

Example 4 When $n = 6$

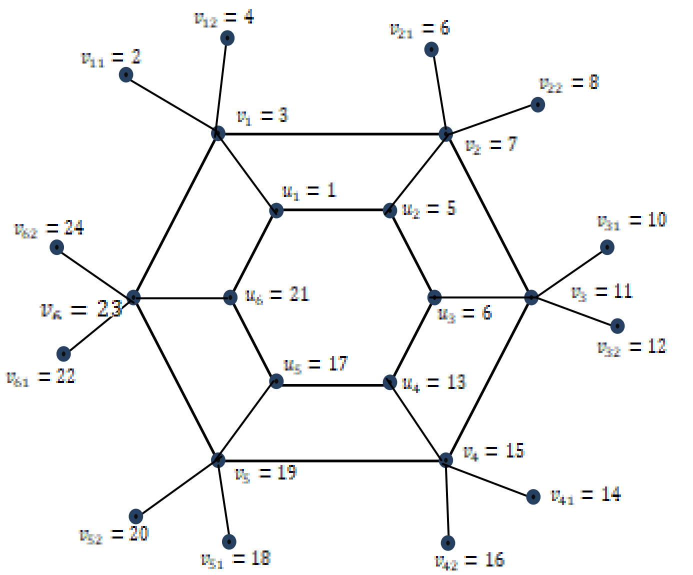

Figure 6: The prime labeling method of the graph obtained by attaching $K_{1,2}$ at each external vertex of the circular ladder graph when $n = 6$.

Theorem 6: If $n + 1$ is prime, then scorpion graph $S_{(2p,2q,r)}$has a prime labeling. Moreover, this prime labeling can be realized with top row labels from left to right, $1, 2, \ldots, n,$ and bottom row labels from left to right, $n + 2, n + 3, \ldots, 2n, n + 1$, where $n = p + q$ and $p \geq 1, q \geq 2, r \geq 2$.

Proof: By using Theorem 1 in [1], vertices have prime labeling, which are in the head and body of the scorpion, when $n + 1$ is prime. Let the starting vertex in the tail is placed at between n and $n + 1$ vertices, and we claim that the following vertex labeling (Figure 3) gives a prime labeling:

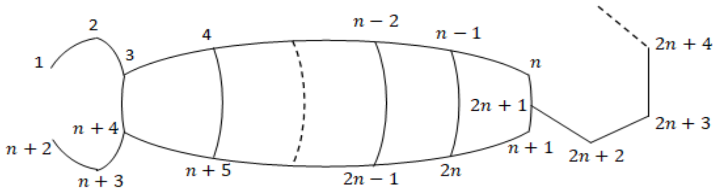

Figure 7: Prime labeling of a scorpion graph when $n + 1$ is prime

Since vertices have prime labeling which are in the head and body of the scorpion, it suffices to check only the vertex labels arising from the endpoints of the following particular edges, the vertical edge connecting vertex labels n and $2n + 1$, and the vertical edge connecting vertex labels $n + 1$ and $2n + 1$. Let $\gcd(n, 2n + 1) = d_1$, then $d_1 | n$ and $d_1 | (2n + 1)$, that implies $d_1 | (n + 1)$. Since d_1 divides both n and $n + 1$, the only possible value for d_1 is 1 (n and $n + 1$ are consecutive positive integers).

Let $\gcd(n + 1, 2n + 1) = d_2$, then $d_2 | (n + 1)$ and $d_2 | (2n + 1)$, that implies $d_2 | n$. Since d_2 divides both n and $n + 1$, the only possible value for d_2 is 1 (n and $n + 1$ are consecutive positive integers). Also, all vertices in the tail are relatively prime because of consecutive positive integers. Thus, the scorpion graph is a prime graph whenever $n + 1$ is prime.

Theorem 7: If $2n + 1$ is prime, then the Scorpion graph $S_{(2p,2q,r)}$ has a consecutive cyclic prime labeling with the value 1 assigned to vertex V_1, where $n = p + q$ and $p \geq 1, q \geq 2, r \geq 2$.

Proof: This proof is the same as the above proof. Here, we only consider the starting point of the tail in the scorpion graph. By using Theorem 2 in [1], vertices have prime labeling, which is in the head and body of the scorpion when $2n + 1$ is prime. Let the starting vertex in the tail is placed in between n and $n + 1$ vertices, and we claim that the following vertex labeling gives a prime labeling:

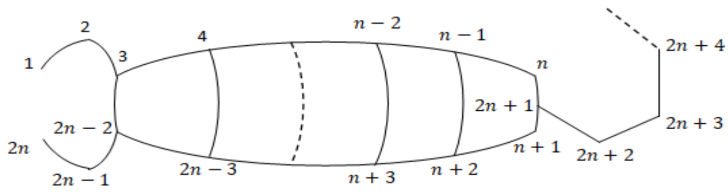

Figure 8: Prime labeling of a scorpion graph when $2n+1$ is prime

Since vertices have prime labeling, which is in the head and body of the scorpion, it suffices to check only the vertex labels arising from the endpoints of the following particular edges: the vertical edge connects vertex labels n and $2n + 1$, and the vertical edge is attaching vertex labels $n+1$ and $2n + 1$.

Let $\gcd(n, 2n+1) = d_1$. Then $d_1|n$ and $d_1|(2n+1)$, implies $d_1|(n+1)$. Since d_1 divides both n and $n+1$, the only possible value for d_1 is 1 (n and $n+1$ are consecutive positive integers).

Let $\gcd(n+1, 2n+1) = d_2$, then $d_2|(n+1)$ and $d_2|(2n+1)$, that implies $d_2|n$. Sinced_2 divides both n and $n+1$, the only possible value for d_2 is 1 (n and $n+1$ are consecutive positive integers). Also, all vertices in the tail are relatively prime because of consecutive positive integers. Thus, the Scorpion graph is a prime graph whenever $2n+1$ is prime.

3. RESULTS AND DISCUSSION

The field of graph theory plays a vital role in discrete mathematics. Graph labeling is one of the important areas of graph theory. Here we present prime labeling of newly constructed graphs in Theorem 1 to Theorem 7. In Theorem 1 and Theorem 4, we discuss the prime labeling method of graph G obtained by replacing every edge of a star graph $K_{1,n}$by the tripartite graph $K_{1,m,1}$. Using the first three examples, it has been shown that the resulting graph has prime labeling when $m = 2,3,4$ and 5 (*Fig.2, 3, and 4*). In addition to that, the stripe blade fan graph $F_{1,3,m}$can be occurred when $n = 3$ in $K_{1,n}$.Further, the following conjecture can be obtained for the stripe blade fan graph.

Stripe Blade Fan graph conjecture: Stripe blade fan graph $F_{1,3,n}$ has prime labeling for all finite $n \in \mathbb{Z}^+$. It has been noticed that once we label vertices with an integer from 1 to n with 1 assigning to the center vertex. The integers between 2 and $m = 3n+4$ for $n \in \mathbb{Z}^+$, there is always a prime number in each one-third interval. Then that number can be assigned to the end of the fan blade. Since other numbers are relatively prime with these prime numbers, the stripe blade fan graph is a prime labeling graph.

Furthermore, another newly constructed graph was introduced by taking the union of circular ladder graph and star graph of $K_{1,2}$. Prime labeling of this graph is given by Theorem 5. It has been illustrated for the case when $n = 6$ by figure 6. Finally, we had proved that the scorpion graphs $S_{(2p,2q,r)}$ have prime labeling when $n + 1$ and $2n + 1$ are prime, where $n = p + q$ and $p \geq 1, q \geq 2, r \geq 2$. It is very interesting to investigate prime labeling of the graphs that have structures of some insects and small animals in the real world.

CONCLUSION

Prime labeling is the most valuable area of graph labeling with various applications. The prime numbers and their behavior are of great importance as prime numbers are scattered, and there are arbitrarily large gaps in the sequence of prime numbers. It has been shown that newly constructed graphs obtained by replacing every edge of a star graph with the tripartite graph $K_{1,m,1}$ for $m = 2,3,4,$ and 5. According to the given results, the graphs obtained by replacing every edge of a star graph $K_{1,n}$ by $K_{1,m,1}$ for $m = 2,3,4,$ and 5 are prime graphs where $n \geq 1$. Also, obtained a prime labeling method for the graphs obtained by attaching $K_{1,2}$ at each external vertex of the circular ladder graph is more difficult when $n \geq 3$ and $n \equiv 1(mod\ 3)$. As a future work, we are planning to generalize these results for scorpion graphs that have walking legs. We would also further investigate additional structures which have prime labeling.

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