RESEARCH ARTICLE

GRAPH THEORY APPLICATIONS FOR GEM INDUSTRY IN SRI LANKA

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ABSTRACT

Sri Lanka is renowned globally for its high-quality gems and its reputation as a premier center for gem cutting and finishing. Despite the industry's significance in revenue generation, various operational challenges persist, including gem mining, cutting and polishing, quality assessment, and supply chain management. This study proposes a novel approach utilizing graph theory to address these challenges. The primary objective is to explore and identify optimal solutions to enhance the efficiency of these challenges within the gem industry through the application of graph theory.

Keywords: *Graph Theory, Gem Industry, Gem Quality Assessment, Similarity Graph, Supply Chain*

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1. INTRODUCTION

Sri Lanka has a unique place in the world of gem mining based on the territory which has made a significant contribution to the economic development. (Sandarenu & Gnanaratne, 2020). Sri Lanka, historically renowned for its exquisite gems, continues to be a leading global supplier of precious stones, particularly blue sapphires. The island's rich gemological heritage, dating back over 2000 years, has played a significant role in its economic development. With a unique geological formation that fosters the formation of

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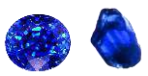
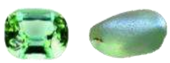
high-quality gemstones, regions like Ratnapura have become central to the gem industry, contributing significantly to both local and international markets. Sri Lanka supplies nearly 75 types of gemstones to the global market. (Sri Lanka Export Development Board). Sri Lankan mines produce a wide range of stones.

Table 1 - Different types of Gems

Gem Type Cut & Rough	Gem Name English/Sinhala	Gem Type Cut & Rough	Gem Name English/Sinhala
	Blue Sapphire නිල් මැණික්		Emerald මරකත
	Yellow Sapphire පුෂ්පරාග		Garnet ගෝමේධ
	Spinel කිරින්චි		Tourmaline තෝරමල්ලි
	Ruby රතු මැණික්		Cat’s eye වෛරෝඩි

The Sri Lankan gem industry plays a crucial role in the country's economy, providing employment and contributing significantly to export revenue. Although the Sri Lankan gem industry is steeped in tradition and globally recognized for its exceptional gemstones, its operational aspects still rely heavily on manual practices and expert judgment. Despite the industry's growth and global reputation, several operational challenges persist, particularly in gem quality assessment, emergency safety exits in mining, and optimizing the gemstone supply chain. The application of advanced mathematical models and computational methods remains limited, especially in process optimization and decision support.

Graph theory is a fundamental area of mathematics with diverse applications across multiple fields. (Rohini Gore et al.,2025). It offers powerful tools to model and solve complex real-world problems by representing systems as networks of nodes and edges. Its utility has been proven across fields such as logistics, social networks, biological systems, and information flow (Sharmila et al., 2023). In particular, clustering algorithms and shortest path models have shown potential in streamlining network operations, which can be effectively applied in the gem industry.

For instance, clustering techniques such as the Highly Connected Subgraph (HCS) algorithm allow for the grouping of similar items based on shared characteristics. This approach has been widely used in network analysis to identify communities or clusters (Chen & Liu, 2022). Within the gem industry, these models can assist in quality assessment and pricing, enabling systematic classification of gemstones based on multiple attributes like color, clarity, and cut.

In the context of mine safety, shortest path algorithms such as Dijkstra's algorithm provide reliable solutions for identifying optimal escape routes in underground environments (Li et al., 2013; Wang & Wu, 2008). These methods are especially relevant in gem mining in Sri Lanka, where traditional excavation techniques still dominate, and rapid response strategies are crucial during emergencies.

Moreover, the Ford-Fulkerson algorithm addresses the maximum flow problem, which is central to optimizing supply chains. In gem trading, where goods pass through a multi-tiered network of miners, dealers, polishers, and exporters, maximizing flow while adhering to capacity constraints can significantly boost efficiency and profits (Ford & Fulkerson, 1956; Rifan & Jayaratne, 2017).

Despite their clear relevance, only a few studies have specifically applied these mathematical tools within the Sri Lankan gem industry. This gap highlights the need for interdisciplinary research to leverage graph-theoretical models in solving practical challenges related to gemstone classification, safety management, and distribution networks. The present study contributes to this space by applying such models across three critical areas of the industry.

This research aims to address these challenges by applying graph theory and its applications to optimize processes within the gem industry, thereby enhancing operational efficiency and safety. Specifically, the study focuses on developing structured approaches for gem pricing, finding shortest paths for safety exits, and maximizing the gemstone distribution through the supply chain network.

2. MATERIAL AND METHODS

2.1 Gem Quality Assessment

This section outlines the step-by-step approach used to construct graphs based on various attributes of gemstones and group them based on their similarities.

2.1.1 Data Collection

The initial phase involved gathering relevant data from reputable sources. Details of Value factors of gemstones were obtained from 'GemGuide' article which was originally published in 2016 by researcher of Gemological Institute of America (GIA) and cut expert, Al Gilbertson. Price tables/Calculation tables were sourced from National Gems and Jewellery Authority.

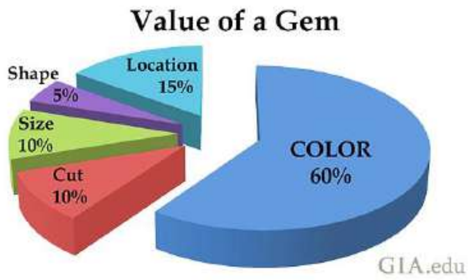


Figure 1 - Value factors of gemstones according to the article "GemGuide",2016

2.1.2 Similarity Graphs and Network Construction

Using the collected data, separate similarity graphs were constructed based on each attribute of gemstones. Nodes in each graph correspond to gemstones, and edges represent similarity between pair of gems. Then, merge the all similarity graphs to create the Similarity network. This can be done by connecting gems to their associated similarities based on the relationships obtained. In similarity network, the weight of each edge indicates the number of similarities of selected two gemstones.

2.1.3 Cluster Analysis

Graph clustering is powerful tool for uncovering patterns, communities, and structures within complex networks. They help partition nodes in a graph into cohesive groups (clusters) based on their common characteristics. In here, Highly Connected Subgraph (HCS) Clustering Algorithm was applied to the similarity network obtained earlier.

2.1.4 HCS Clustering Method/Steps

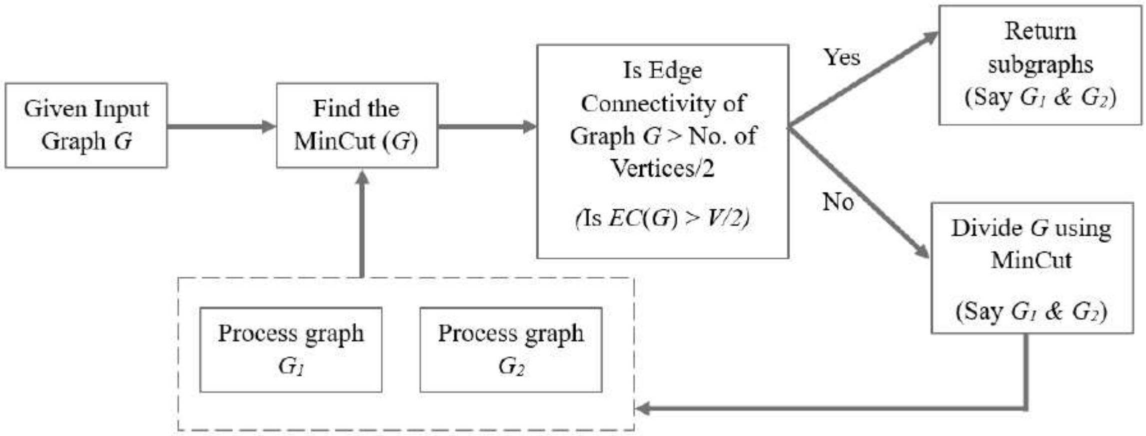


Figure 2 - Steps of HCS Clustering Algorithm

2.2 Finding the shortest path for safety exits in emergency situations

This section outlines the step-by-step approach used to Find the shortest path in gem mines for safety exits in case of emergency.

2.2.1 Data Collection

In this study, a hypothetical gem mine layout was used to model emergency escape paths and demonstrate the application of Dijkstra's algorithm for shortest path identification. Although real-time structural datasets of gem mines in Sri Lanka are limited, the model was developed based on first-hand information obtained from experienced mine workers in the Ratnapura region. Additional references on gem mining techniques and other relevant data were gathered from websites of the Gemological Institute of America (GIA) and National Gem and Jewellery Authority of Sri Lanka.

2.2.2 Construction of the mine network using the graph theory principles

Gem mine network/graph was constructed based on the structure of the gem mine and using gathered information. To reflect a typical mining scenario; the one and only exit point of the mine is represented by vertex 1 (highlighted in green in the graph), and the remaining vertices/nodes (2, 3, 4, …, 15) in graph correspond to each junction of two or more tunnels/routes, and edges represent the route or tunnel between two junctions. The weight assigned to each edge indicates the physical distance in meters between two junctions.

2.2.3 Applying Dijkstra's algorithm for Gem mine network

Dijkstra's algorithm is a popular algorithm for solving many single-source shortest path problems having non-negative edge weight in the graphs i.e., it is to find the shortest distance between two vertices on a graph. It was conceived by Dutch computer scientist Edsger W. Dijkstra in 1956.

The algorithm maintains a set of visited vertices and a set of unvisited vertices. It starts at the source vertex and iteratively selects the unvisited vertex with the smallest tentative distance from the source. It then visits the neighbors of this vertex and updates their tentative distances if a shorter path is found. This process continues until the destination vertex is reached, or all reachable vertices have been visited.

2.2.4 Web application for Dijkstra's algorithm

When network becomes more complex, manual calculations take considerable time to calculate the shortest distances as well as the shortest paths, considering one node at a time. Therefore, this study has used some Web application called Dijkstra Solver.

Dijkstra Solver Web Application - We have to input all the edges with two ending nodes and their respective edge weights. Also, we have to input our source vertex (current location) and the vertex we considered as our safety exit at that time. Not only that, but we should have to put a blue tick on the box in front of the word directed when graph is directed.

2.3 Optimizing Supply Chain in Gem Industry

This section outlines the step-by-step approach used to optimize the Supply Chain in Gem Industry.

2.3.1 Data Collection

The initial phase involved gathering relevant data from reputable sources. Rifan and Jayaratne (2017) conducted a detailed evaluation of the colored gemstone supply chain in Sri Lanka, identifying key factors such as miners, collectors, brokers, cutters, polishers, and exporters. These insights provide a strong foundation for constructing the supply chain network.**2.3.2 Construction of the supply chain network using the graph theory principles**

Every product has a Supply Chain (SC) consisting of several integrated processes, extending from raw materials to end products and beyond end use. In the supply chain of the gem industry, large quantities of gemstones are exchanged between traders on a daily basis. All parties in the supply chain aim to earn better profits by optimizing the distribution of gems in the supply chain.

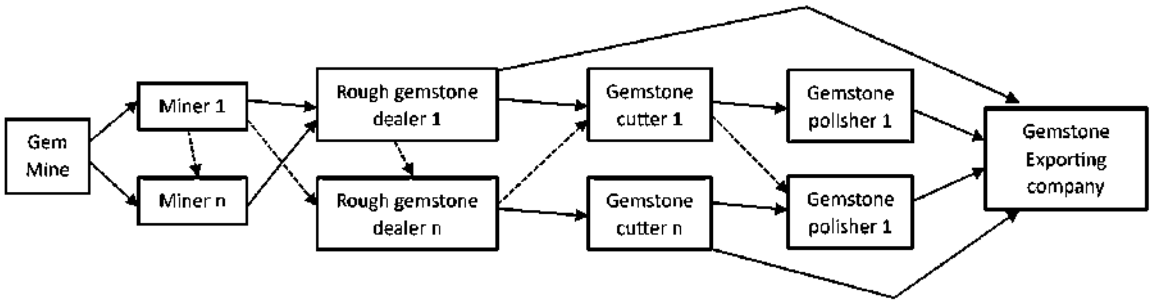


Figure 3 - Flow Chart of Supply Chain Network in Gem Industry

By using above flow chart, supply chain network of gem industry was constructed. Source represents the starting point where the gemstones are mined. (Gem mine). Intermediate Nodes represent intermediate and processing persons such as gem dealers, gem cutters and polishers. Sink represents the export company where the gemstones are sold to foreigners. The edges between nodes represent the transportation routes and weight of an edge/ capacity of an edge is representing the maximum number of gemstones that can be transported through that route.

2.3.3 Applying Ford-Fulkerson Algorithm for Maximum Flow Problem to Gem supply chain network

The Ford-Fulkerson algorithm is a widely used algorithm to solve the maximum flow problem in a flow network. The maximum flow problem involves determining the maximum amount of flow that can be sent from a source vertex to a sink vertex in a directed weighted graph, subject to capacity constraints on the edges.

The algorithm works by iteratively finding an augmenting path, which is a path from the source to the sink in the residual graph, i.e., the graph obtained by subtracting the current flow from the capacity of each edge. The algorithm then increases the flow along this path by the maximum possible amount, which is the capacity of the edges along the path.

2.3.4 Objectives and Constraints

Each edge in the network has a capacity, denoted as $C_{i,j}$ which represents the maximum number of gemstones that can be transported from node *i* to node *j*. The flow, denoted as $X_{i,j}$ represents the actual number of gemstones transported along that edge. The objective is to determine the maximum number of gemstones that can be transported from the mines (source) to the export company (sink) while respecting the capacity constraints on the edges, through the intermediate nodes using the Ford-Fulkerson algorithm. The flow must satisfy the following conditions;

1. Capacity Constraint: $X_{i,j} \leq C_{i,j}; \ \forall_{i,j}$
2. Non-Negativity Constraint: $X_{i,j} \geq 0; \forall_{i,j}$
3. Kirchoff's Law: $\sum_{K=1}^{n} X_{i,k} = \sum_{K=1}^{m} X_{k,i}$; $\forall_i$ except for the source and the sink.

2.3.5 Web Application for Ford-Fulkerson algorithm

When network becomes more complex, manual calculations take considerable time to calculate the maximum flow from source to sink. Therefore, this study has used a Web application in Graph Online website. Here first we need to draw our graph by using the given tool base. Then we should also select "Finding Maximum Flow" from the column labeled as algorithms and mark our source and sink on our graph. Then it shows visually us the maximum flow.

3. RESULTS AND DISCUSSION

3.1 Gem Quality Assessment

3.1.1 Data Collection

Let's consider twelve blue sapphire gems: *A, B, C, …, L*. We'll use a weighted undirected graph to represent their similarities: (*Note that in here, the same type gemstones are considered.)

Table 2 - Representation of Gem Attributes

Gem Stone	Color	Clarity	Carat weight	Cut	Shape	Market Demand
A	First color (3)	slightly included	1.7 (1-2 range)	Good	Round	High
B	Medium color (1)	Eye clean	3.8 (3-4 range)	Better	Ovel	High
C	Light color	heavily included	0.9 (<1)	Excellent	Pearl	Low
D	Medium color (1)	slightly included	2.4 (2-3 range)	Good	Heart	Law
E	Medium color (2)	heavily included	5.3 (5-10 range)	Excellent	Emerald	Law
F	First color (1)	slightly included	3.1 (3-4 range)	Better	Ovel	High
G	Light color	Eye clean	4.2 (4-5 range)	Good	Emerald	Low
H	Medium color (3)	heavily included	2.7 (2-3 range)	Excellent	Heart	Law
I	First color (2)	slightly included	1.9 (1-2 range)	Good	Round	High
J	First color (3)	heavily included	10.4 (10<)	Excellent	Pearl	High
K	Medium color (1)	Eye clean	3.1 (3-4 range)	Better	Ovel	High
L	Medium color (2)	Eye clean	0.81 (<1)	Good	Round	High

3.1.2 Similarity Graphs and Network Construction

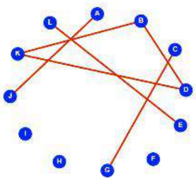


Figure 4 - Color Similarity Graph

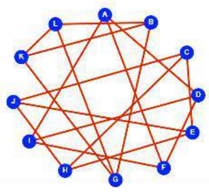


Figure 5 - Clarity Similarity Graph

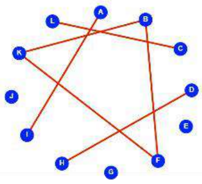


Figure 6 - Weight Similarity Graph

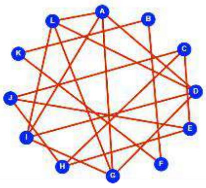


Figure 7 - Cut Similarity Graph

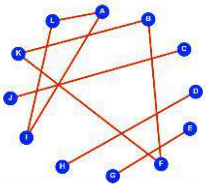


Figure 8 - Shape Similarity Graph

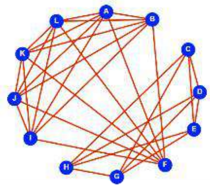


Figure 9 - Market Demand Similarity Graph

3.1.3 Applying HCS Clustering Algorithm

Step 01: First, merge the all-similarity graphs and create the Similarity network/graph

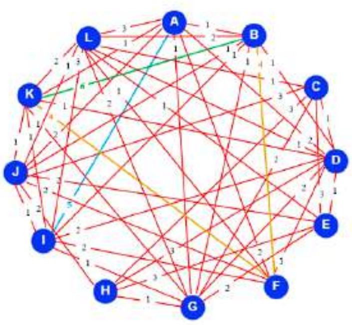


Figure 10 - Graph G

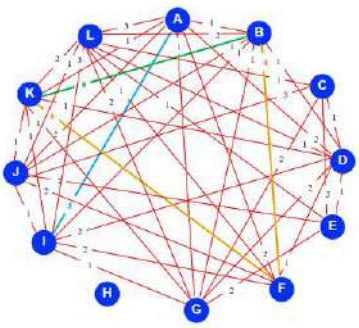


Figure 11 - MinCut(G)

$$MinCut\ (G) = \{(H, C), (H, D), (H, E), (H, G), (H, J)\}$$
$$Total\ edge\ weight\ of\ MinCut = 12$$

Step 02: Check whether, Is $EC(G) > V/2$

- $Edge\ Connectivity\ (G) = EC(G) = |\ MinCut(G)\ |$
  $= |\ \{(H,C), (H,D), (H,E), (H,G), (H,J)\}\ |$
  $= 5$
- $V/2 = 12/2 = 6$
- $EC(G) \ngtr V/2$, *therefore G is not a highly connected subgraph*

Step 03: Since $EC(G) \ngtr V/2$, We divide *G* using *MinCut*. (let's say G_1 and G_2)

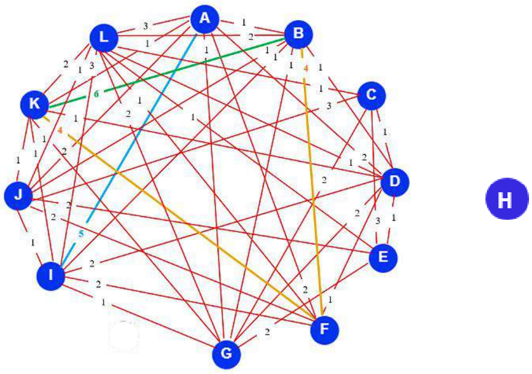


Figure 12 - Graph G1 and G2

Step 04: Repeat above four steps. After repeating all steps, we get following output.

Input: Obtained Similarity network

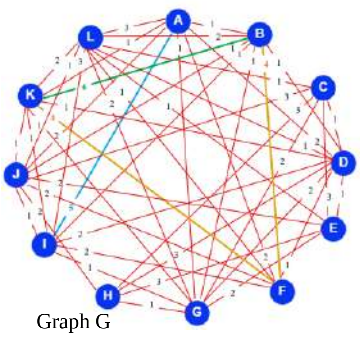


Figure 13 - Similarity Network

Output: Resultant Clusters after applying HCS Clustering Algorithm

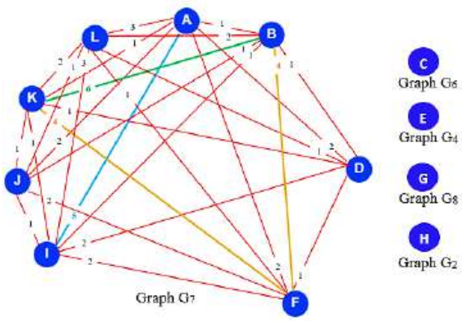


Figure 14 - Resultant Clusters

By using Highly Connected Subgraph (HCS) Clustering Algorithm, we can divide our main graph into 5 subgraphs. By dividing it into several subgraphs, we can simplify our main graph and it helps to easily identify gems which has more similarities. It can also be noted that other gemstones have more similarities than *C*, *E*, *G* and *H* gemstones.

Gem *B* and Gem *K* have the maximum number of similarities. (Because of maximum weight). Therefore, it is clear that, for *B* and *K* gemstones are in similar price range. Apart from this, the price range of *F* is quite similar to *B* and *K* gems because it also has more similarities. And also, *A* and *I* gems also have more similarities. Therefore, their price ranges are very similar.

3.1.4. Pricing

Table 3 - Blue Sapphire Calculation table according to color, clarity & weight

Blue Sapphire - Calculation Table									
No	Color	Clarity	Weight (cts) & Value (Rs)						
			1<	1 - 2	2 - 3	3 - 4	4 - 5	5 - 10	10<
1	First Color	Eye clean	7500-15000	15000-60000	30000-60000	60000-80000	80000-100000	100000-300000	300000 & above
		Slightly included	5000-8000	8000-60000	30000-50000	50000-70000	70000-80000	80000-250000	250000 & above
		Heavily included	300-500	500-8000	4000-8000	8000-15000	15000-50000	50000-100000	100000 & above
	First Color (2)	Eye clean	6000-8000	8000-40000	20000-40000	40000-60000	60000-90000	90000-250000	250000 & above
		Slightly included	4500-7000	7000-10000	5000-30000	30000-50000	50000-70000	70000-200000	200000 & above
		Heavily included	250-400	400-3000	1500-6000	6000-13000	13000-40000	40000-90000	90000 & above
	First Color (3)	Eye clean	5000-7000	7000-30000	15000-30000	30000-50000	50000-80000	80000-200000	200000 & above
		Slightly included	3000-6000	6000-9000	4500-20000	20000-40000	40000-60000	60000-150000	150000 & above
		Heavily included	200-350	350-4000	2000-5000	5000-10000	10000-30000	30000-80000	80000 & above
2	Medium colour	Eye clean	6000-8000	8000-25000	12500-30000	30000-40000	40000-70000	70000-150000	150000 & above
		Slightly included	4000-5000	5000-18000	9000-18000	18000-35000	35000-50000	50000-100000	100000 & above
		Heavily included	200-300	300-4000	2000-4000	4000-8000	8000-20000	20000-70000	70000 & above
	Medium colour (2)	Eye clean	4000-6000	6000-10000	5000-25000	25000-35000	35000-60000	60000-100000	100000 & above
		Slightly included	3500-4500	4500-8000	4000-15000	15000-30000	30000-40000	40000-90000	90000 & above
		Heavily included	100-250	250-2000	2000-3000	3000-7000	7000-15000	15000-60000	60000 & above
	Medium colour (3)	Eye clean	2000-3000	3000-9000	4500-20000	20000-30000	30000-40000	40000-90000	90000 & above
		Slightly included	1500-2500	2500-7000	3500-13000	13000-20000	20000-30000	30000-80000	80000 & above
		Heavily included	100-200	200-2000	1000-2000	2000-5000	5000-10000	10000-50000	50000 & above
3	Light colour	Eye clean	11500-2500	2500-8000	4000-18000	18000-25000	25000-30000	30000-80000	80000 & above
		Slightly included	1000-2000	2000-5000	2500-12000	12000-18000	18000-20000	20000-70000	70000 & above
		Heavily included	50-200	200-1000	500-1500	1500-3000	3000-8000	8000-40000	40000 & above

Price range of gemstones can be determined according to table 1 based on their attributes. It is clear that, for B and K gemstones are in similar price range. Price range of B and K gems according to table 1 is: Rs.30,000 – Rs.40,000

3.2 Finding the shortest path for safety exits in emergency situations

3.2.1 Applying Dijkstra's Algorithm

Input: Constructed Gem Mining Network

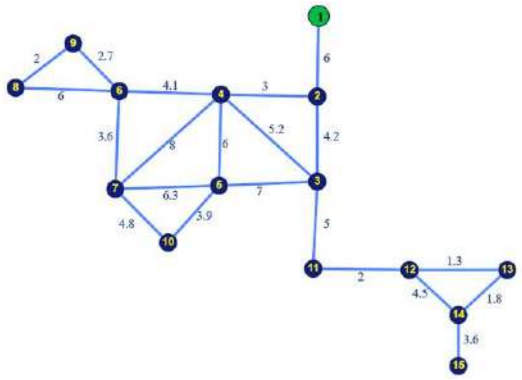


Figure 15 - Constructed Gem Mining Network

Output: Resulting graph with shortest distances (in red color) from node 1 to every other node after applying Dijkstra's algorithm

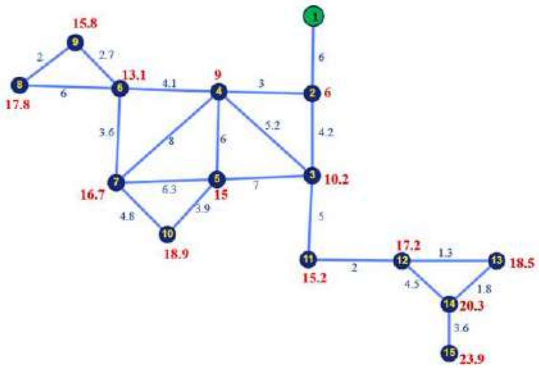


Figure 16 - Resultant graph with shortest distances

3.2.2 Finding Shortest Paths by Using Dijkstra Solver

The following table shows the shortest paths from each and every other node to vertex 1 (safety exit), related to shortest distances that are obtained from this web application. Note that all the distances mentioned here are in meters.

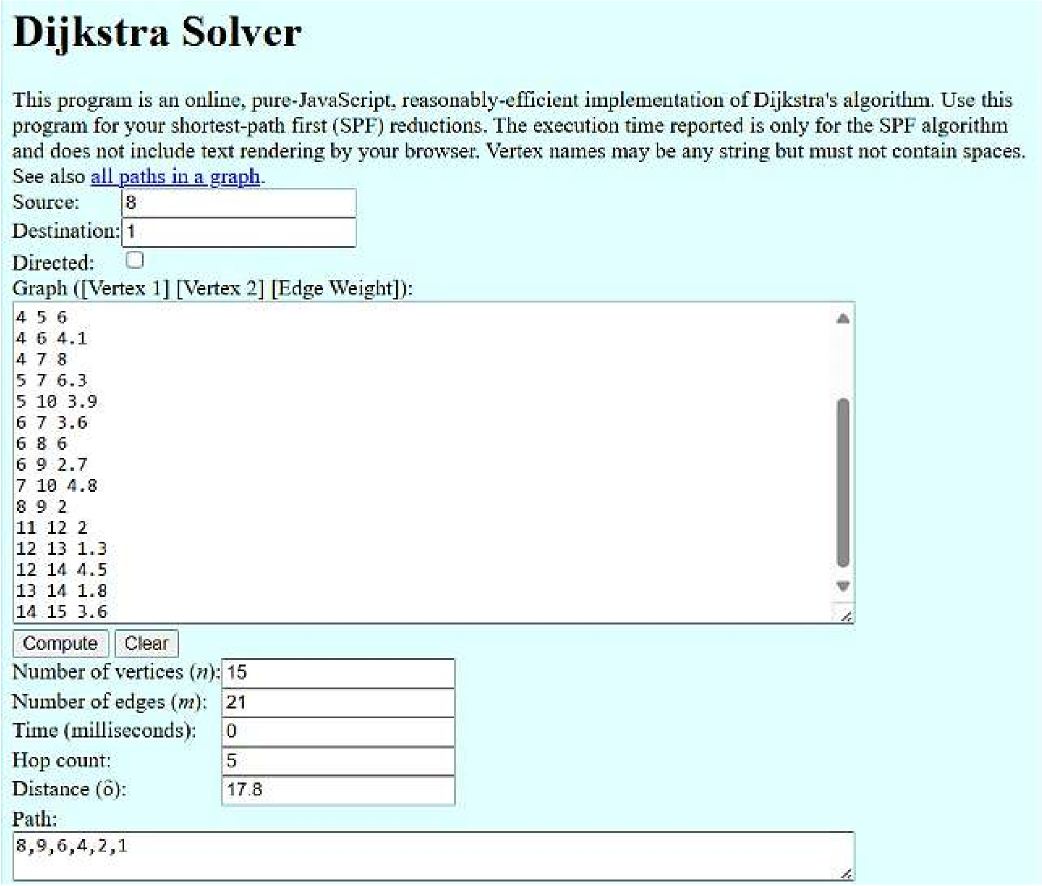


Figure 17 - Interface of Dijkstra Solver Web Application

Table 4 - Shortest Paths and distances from node 1 to every other node

Current location	Shortest path from that location to safety exit	Shortest distance
Vertex 2	2 → 1	6
Vertex 3	3 → 2 → 1	10.2
Vertex 4	4 → 2 → 1	9
Vertex 5	5 → 4 → 2 → 1	15
Vertex 6	6 → 4 → 2 → 1	13.1
Vertex 7	7 → 6 → 4 → 2 → 1	16.7
Vertex 8	8 → 9 → 6 → 4 → 2 → 1	17.8
Vertex 9	9 → 6 → 4 → 2 → 1	15.8
Vertex 10	10 → 5 → 4 → 2 → 1	18.9
Vertex 11	11 → 3 → 2 → 1	15.2
Vertex 12	12 → 11 → 3 → 2 → 1	17.2
Vertex 13	13 → 12 → 11 → 3 → 2 → 1	18.5
Vertex 14	14 → 13 → 12 → 11 → 3 → 2 → 1	20.3
Vertex 15	15 → 14 → 13 → 12 → 11 → 3 → 2 → 1	23.9

3.3 Optimizing Supply Chain in Gem Industry

3.3.1 Applying Ford-Fulkerson Algorithm for Maximum Flow Problem

Input: Constructed Supply Chain Network

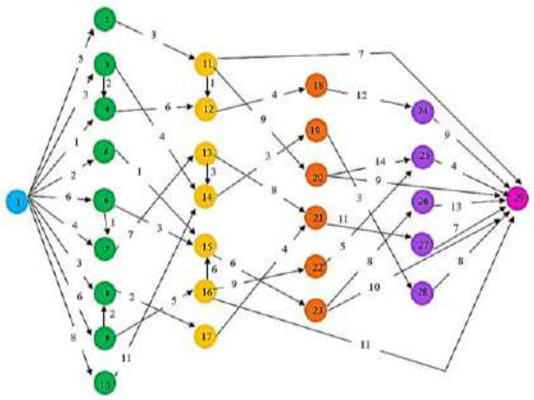


Figure 18 - Constructed Supply Chain Network

Output: Resulting graph with maximum flow (in yellow color) from source to sink.

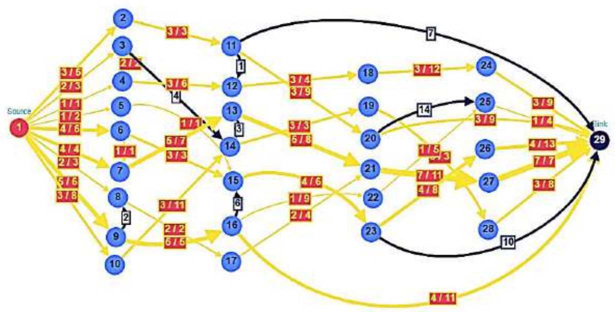


Figure 19 - Resultant graph with maximum flow

3.3.2 Finding Maximum Flow by Using Graph Online Web Application

The following is a resulting graph with Maximum flow (in yellow color) from source to sink. First, we need to open Graph Online Web Application and draw our graph by using the given tool base. Then we should also select "Finding Maximum Flow" from the column labeled as algorithms and mark our source and sink on our graph. Then it shows visually us the maximum flow.

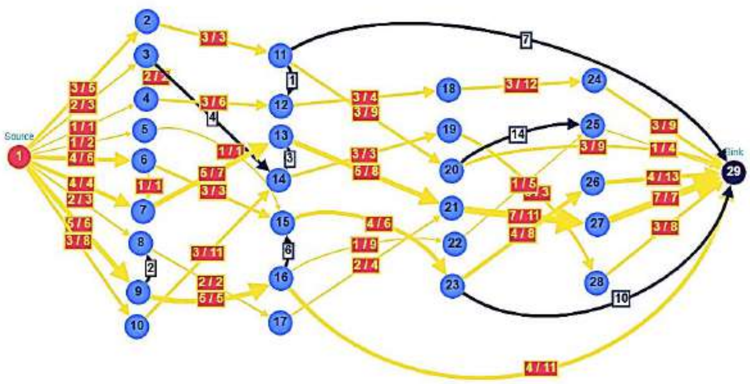


Figure 20 - Resultant graph with maximum flow of Graph Online Web Application

The maximum flow of our Gemstone supply chain is 25.

4. CONCLUSION

This study addresses three critical challenges within Sri Lanka's gem industry by leveraging graph theory. First, the Highly Connected Subgraphs (HCS) clustering algorithm was used to sort gemstones by key similarity attributes, facilitating accurate pricing. Second, Dijkstra's algorithm was utilized to determine the shortest and safest exit routes in emergencies for enhancing miner safety. Finally, the Ford-Fulkerson algorithm optimized the gemstone supply chain by maximizing flow efficiency. The findings of this research not only contribute to a deeper understanding of gem valuation and safety in mining operations but also offer practical solutions for optimizing the gemstone supply chain, benefiting industry stakeholders at all levels.

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